



Informal statistical inference

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Outline

- Background & some findings from my research
- Experiencing sampling variability
- Brief introduction to comparative reasoning and informal inference



Some stories from research

- Teaching the concepts of sampling distributions (Chance, delMas, & Garfield, 2004; Saldanha & Thompson, 2002)
- Concluded: students' conceptual growth hindered by limited understanding and lack of a sense of variability and distribution
- Lack of attention to conceptual development in earlier years



Some stories from research

- Year 11: Drawing conclusions from the comparison of two distributions (e.g., Which battery is better, A or B?) (Pfannkuch, 2006, 2007)
- Was the teacher drawing an inference about the population from the sample?
- On what grounds were students drawing conclusions?
- Students - no concepts of sample, population, sampling variability




Research Question

- How can students be stimulated to start developing a conceptual structure with which they can conceive sampling variability?




Research Lunchtime task [\(www.censusatschool.org.nz\)](http://www.censusatschool.org.nz)



THE UNIVERSITY OF AUCKLAND
FACULTY OF SCIENCE
Department of Statistics

census at school 2005

Data Viewer



DATA

Sample Size (n):

Resampling completed

ANALYSIS

Lunchtime activities: for

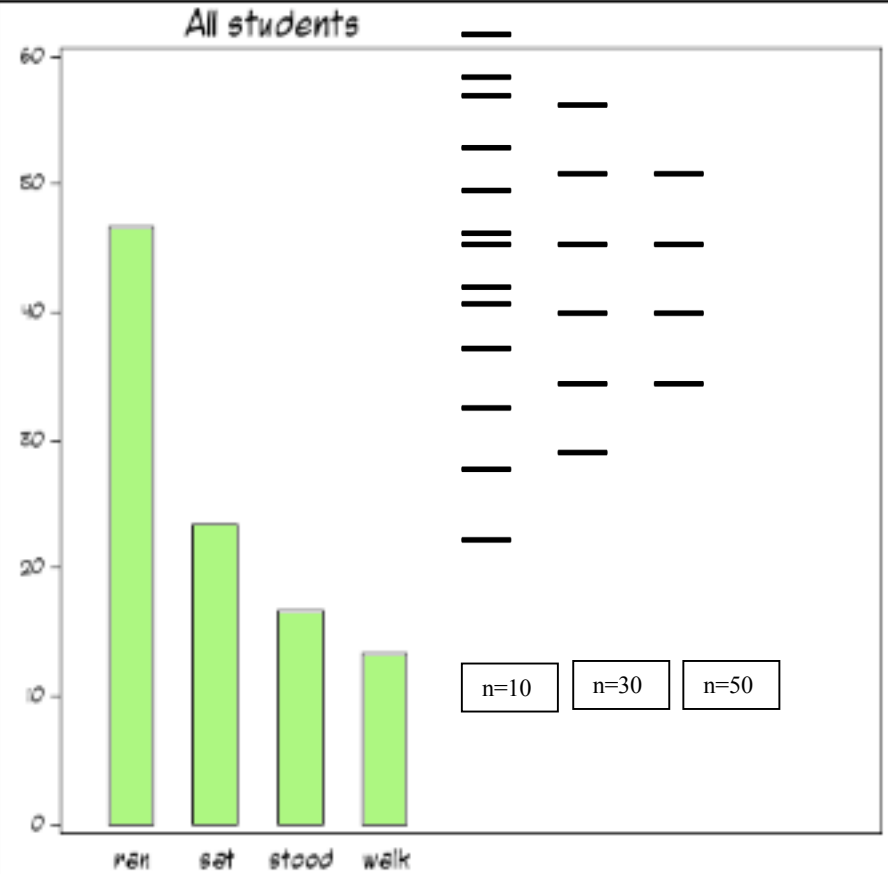
Bar graph

Summary

All students				
ran	sat	stood	walk	n
14	7	5	4	30
47%	23%	17%	13%	100%

- [Change sample settings](#)
- [Export to Excel](#)
- [View data](#)

All students

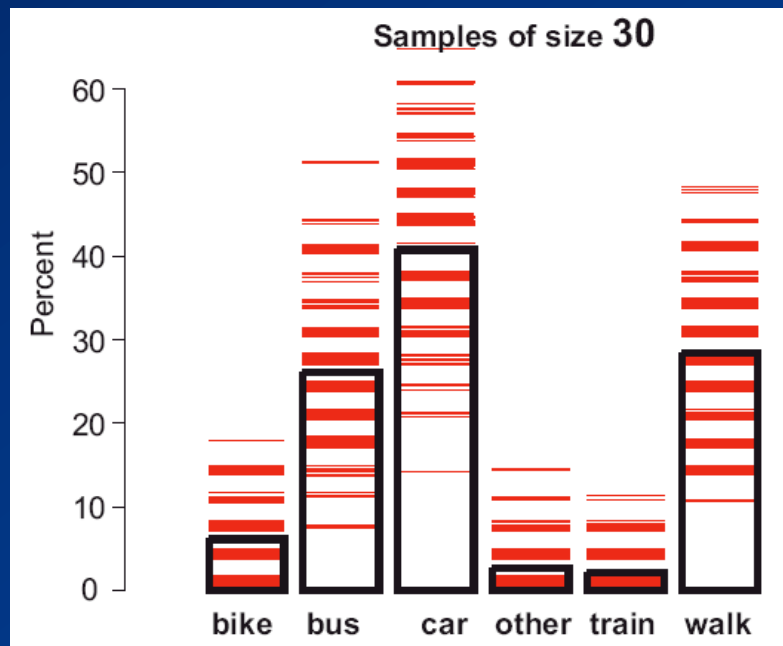
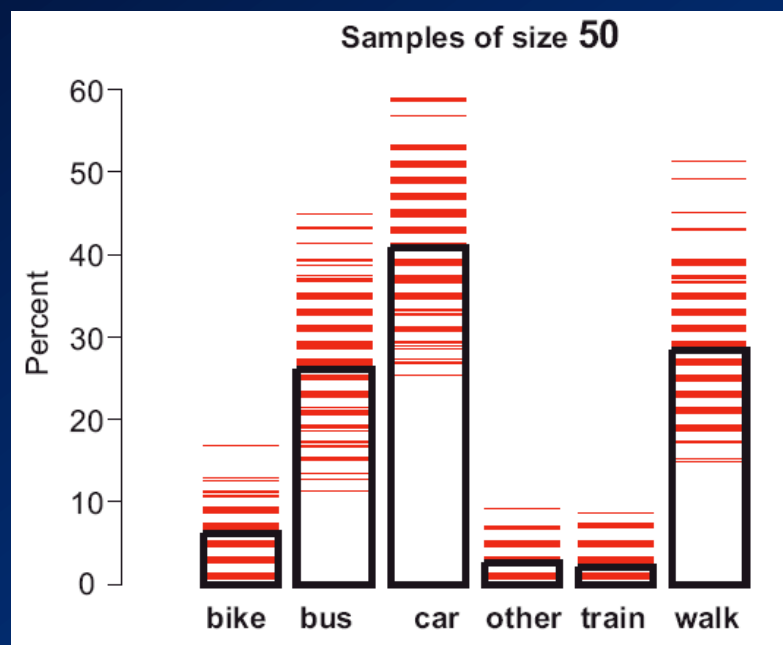
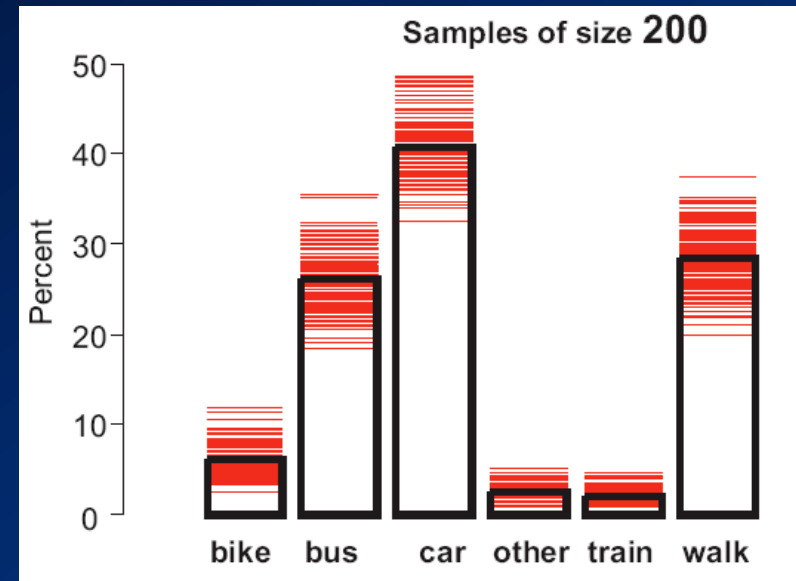
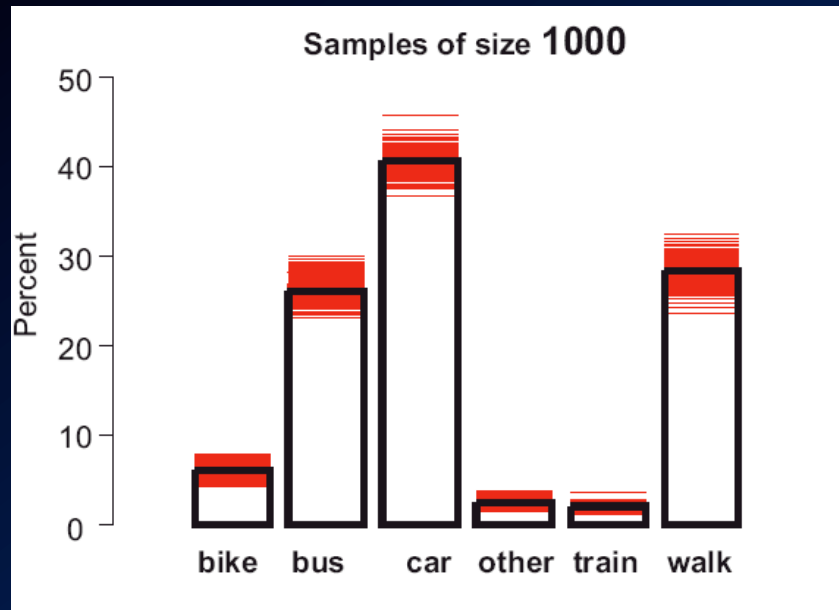




Samples of size 100



“What I see is not quite the way it really is”





Research: Height Task

- Gaining an image of “intuitive confidence interval” for median

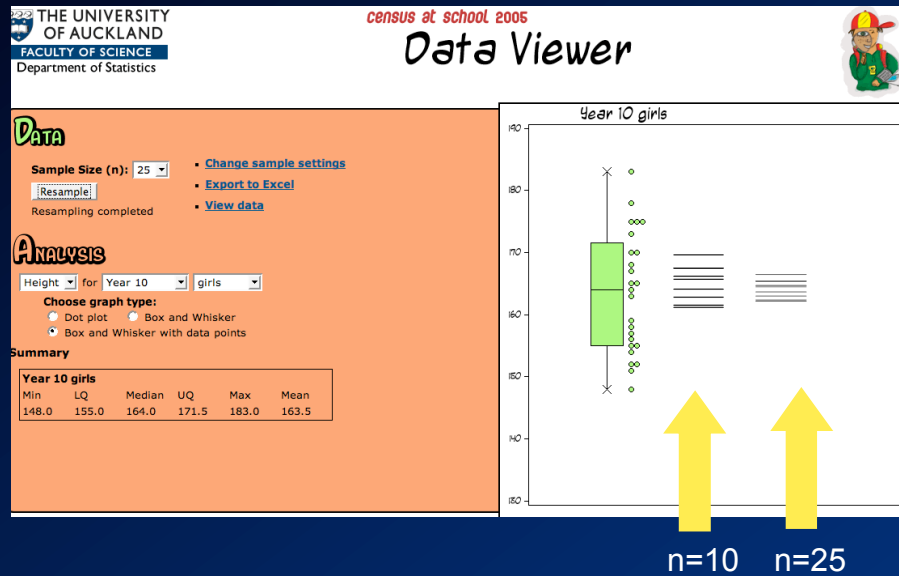
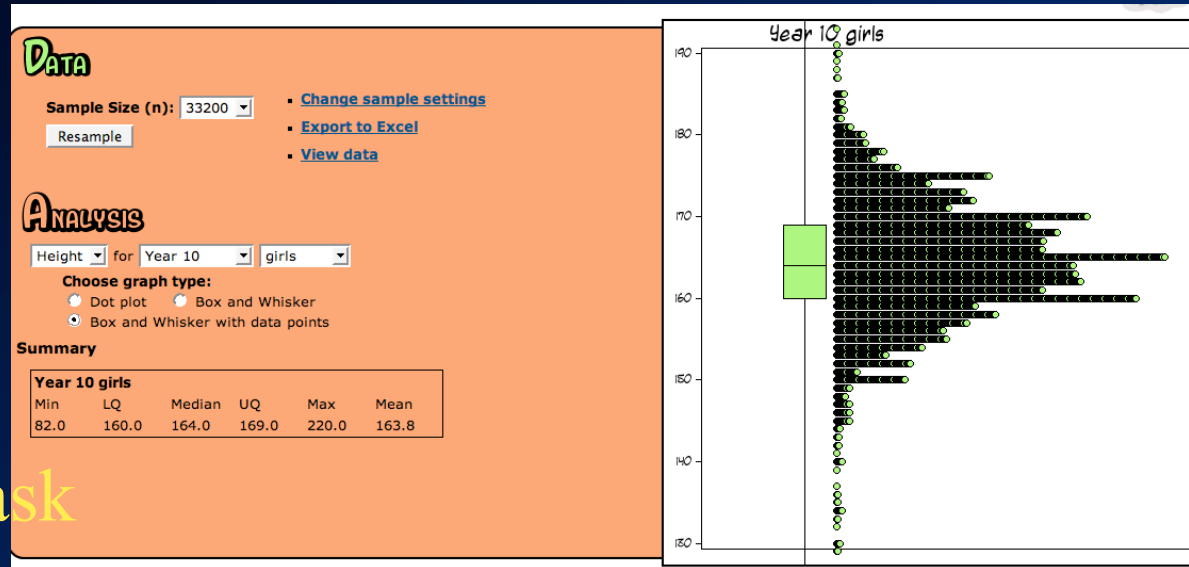




Image of relationship of sample distribution and population distribution

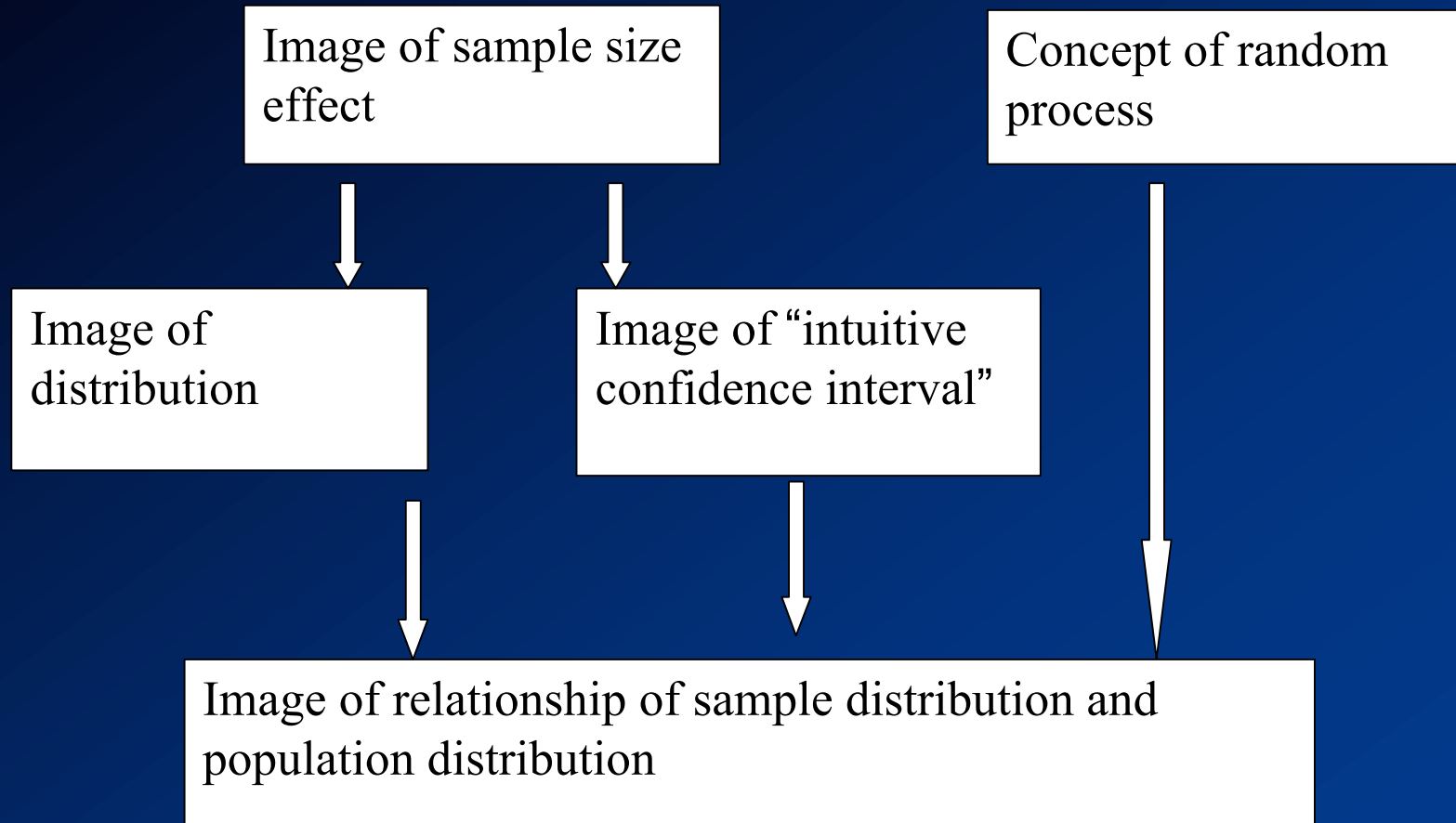


Height Task

- T: Tell me about this whole population.
- S: The median is roughly the same as the sample



Framework for ways of thinking about sampling variability





Some student quotes

- I found it interesting when you only need a sample to work out problems from a large population. It's just really weird and cool at the same time that you just take any sample that is representative and it gives you near enough accurate answers.
- Comparing distributions with different sized sample graphs because the variation of the graph changing each time when the sample was small but having less variation when the sample size got larger.
- The part where we compares and saw all the different medians from different sample sizes. Coz it was really interesting to see it change. Its buzzy...



Experiencing sampling variability

- Hands-on
- Computer generated simulations
- Keep a history of the variability
- Build up concepts of sample, population, sample size effect, distribution, confidence interval etc.
- Making a call: Which is bigger?
 - From class activities and simulations (Pip's workshop) develop “guidelines” or “rules” that allow students to make a claim or informal inference
 - Build up “rules” from level 5 to level 8 towards statistical inference



How do students currently “make a call”?

- Use the **MEDIANS** (Pfannkuch, 2008)



Making the Call – the basic idea

Making the Call – the basic idea

Observed data:

Back in the populations:

“Do μ values tend to be bigger than ν values?”

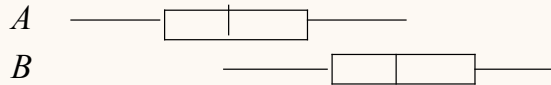
My call is



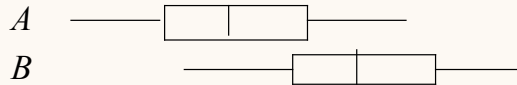
B is bigger



all sample sizes



B is bigger



Claim “B is bigger”
if both sample sizes > 20

Larger random samples have
more information about the
populations they came from.



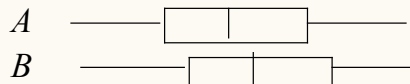
What’s my call here?

Thus, with larger random samples,
we can make the “B is bigger”
call from smaller shifts

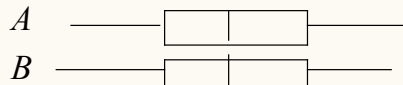


What’s my call here?

But how do we decide?
- depends on educational level of students
- see next page ...



Call “Cannot tell”
unless both samples are huge



Cannot tell

all sample sizes

Warning to teachers: avoid doing this sample with sizes smaller than about 20 in each group. Small samples quite often give rise to unstrange and often very strange box plots. To echo the previous diagram, we get very large distortions -- see plots for samples of size 10 on page 6

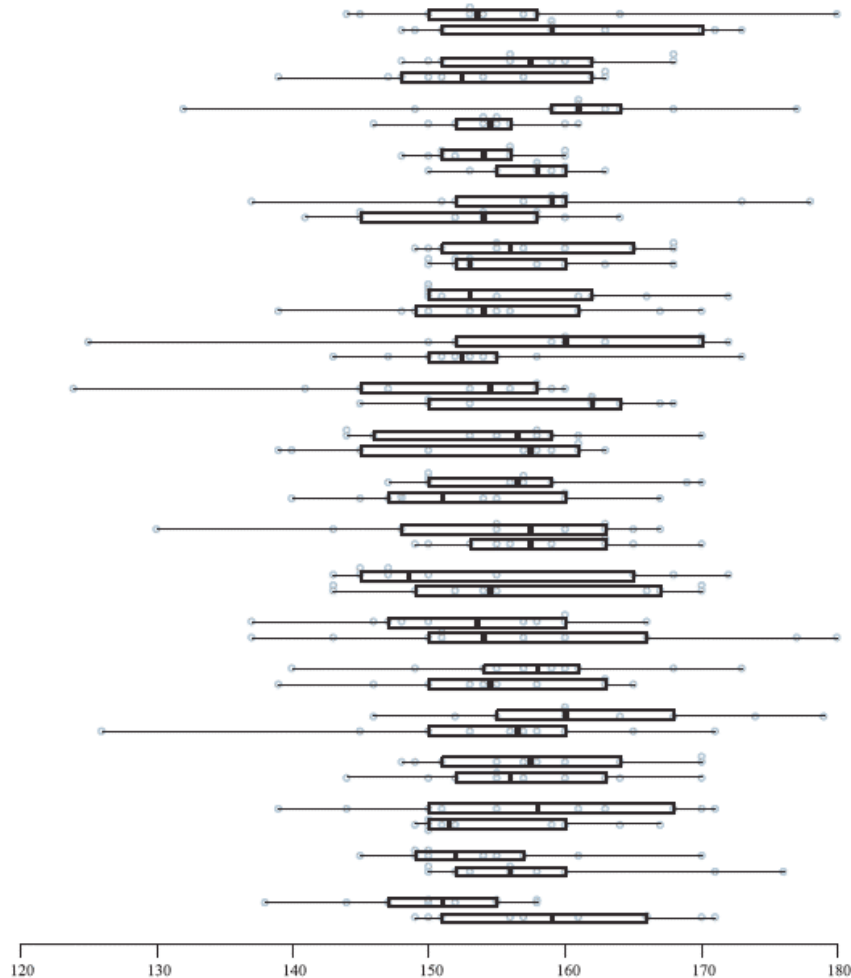


Examples of shifts called purely by sampling

Population distribution



Samples of size 10

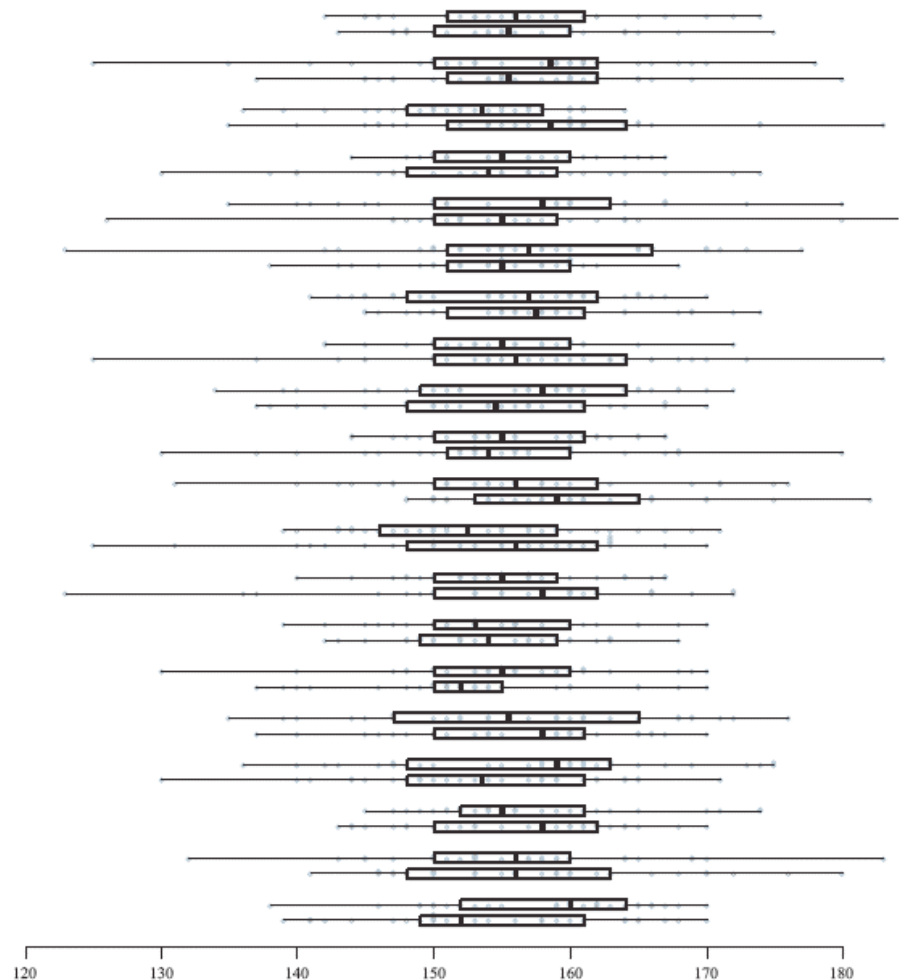


Samples of size 10 shown to demonstrate why we should not be working in this way with such small samples

Population distribution



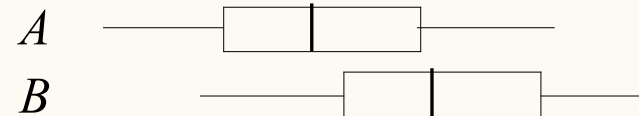
Samples of size 30





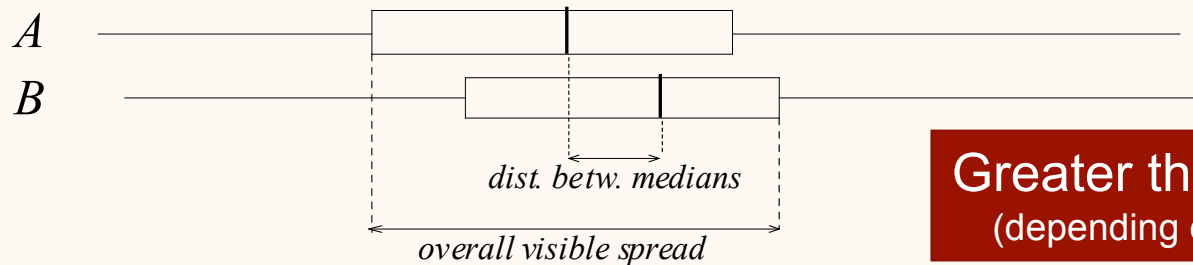
Which is bigger? *(back in the populations)*

Curriculum Level 5: *the 3/4-1/2 rule*



Majority of one to the right of “the great whack” of the other

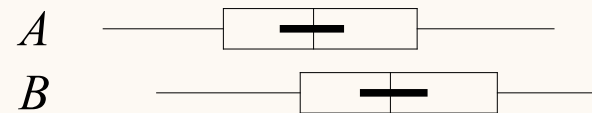
Curriculum Level 6: *distance between medians as proportion of “overall visible spread”*



Greater than 1/3 or 1/5
(depending on sample size)

Curriculum Level 7: *based on informal confidence intervals for the population median*

Make the claim *B tends to be bigger than A* back in the populations



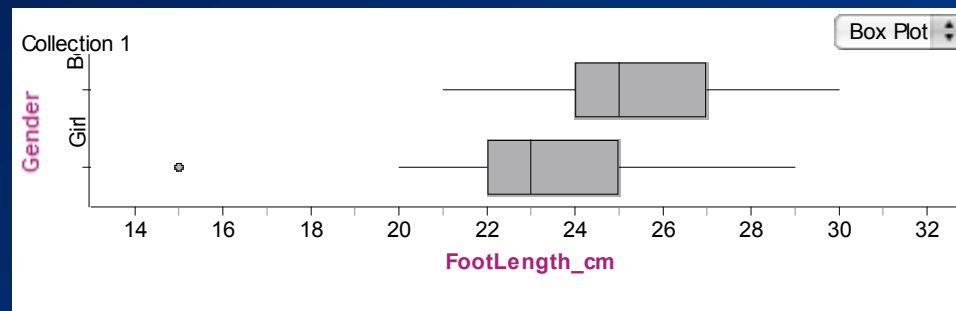
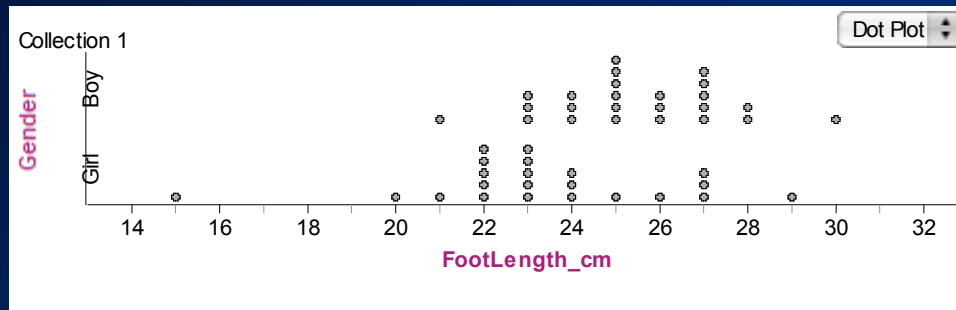
No overlap of constructed intervals



How would we like students to reason?

Investigative Question

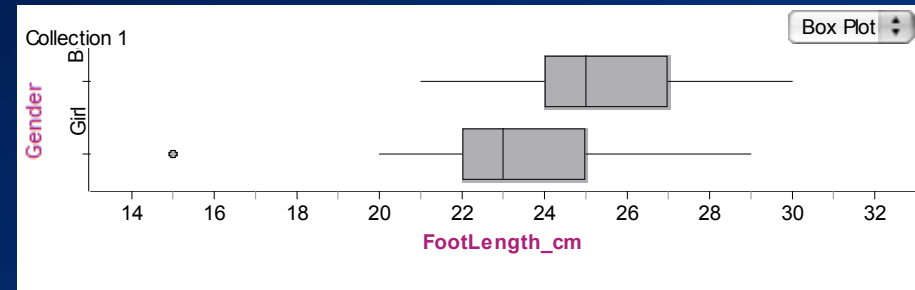
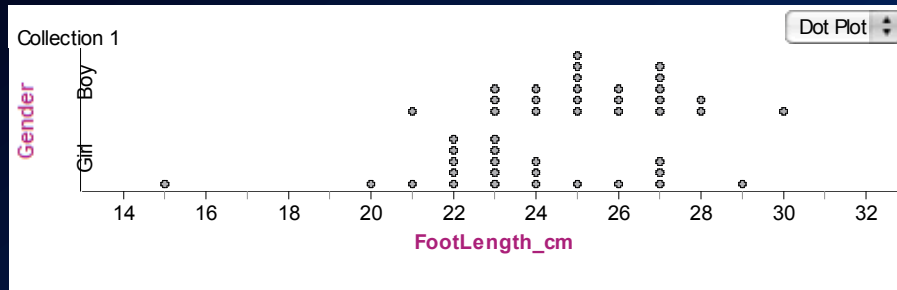
Do 13 year-old NZ boys tend to have bigger right foot lengths than 13 year-old NZ girls?



Two random
Samples from
CensusAtSchool



Overall visual non-numerical comparisons



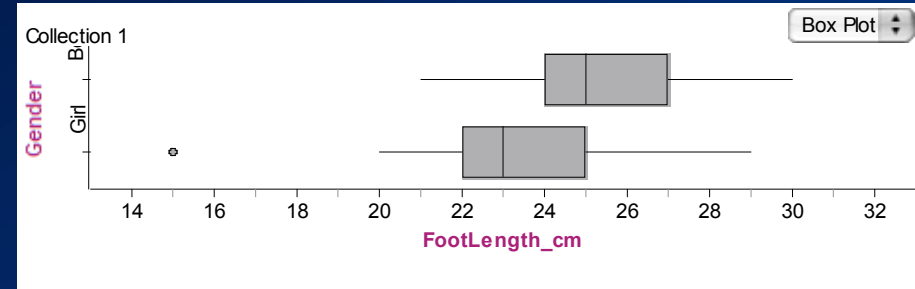
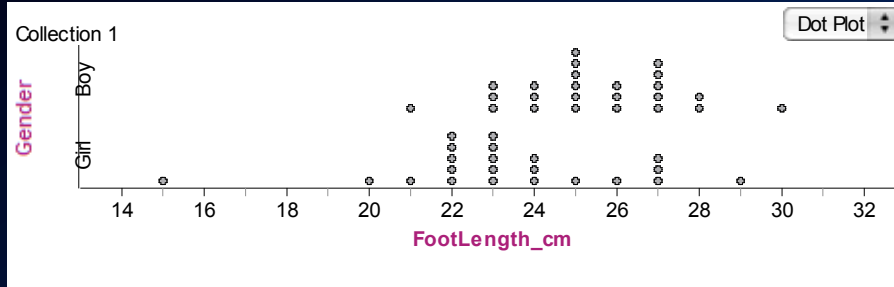
- Overlap
- Shift
- Unusual features



Then consider 8 elements: spread,
shape, summary statistics, explanatory
etc. (Pfannkuch, 2006; 2007)



Spread



I notice (*descriptive*):

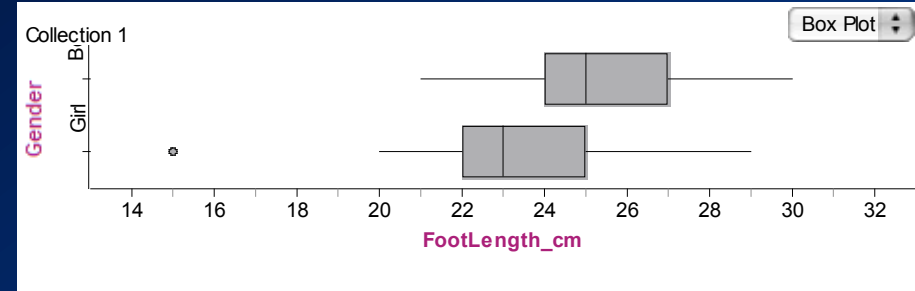
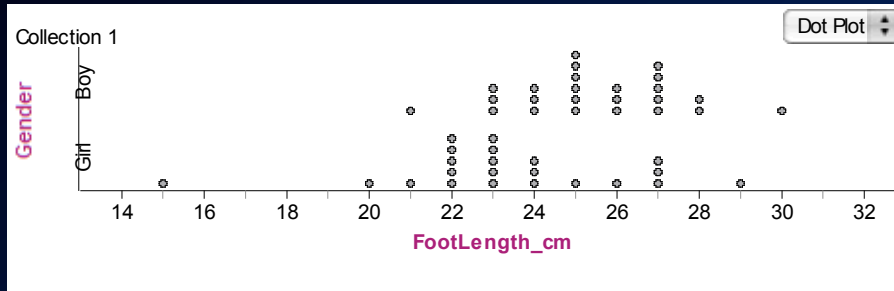
- the middle 50% of the boys have a right foot measuring between 24 and 27cm (IQR = 3cm) whereas the middle 50% of the girls are between 22 and 25cm (IQR = 3cm).
 - This means that the foot lengths for these boys **vary** by about the same amount as these girls' do.

I wonder (*inferential*):

- if boys' and girls' foot length distributions **back in the two populations** have similar variability.
 - I expect so.



Shape



I notice (*descriptive*): :

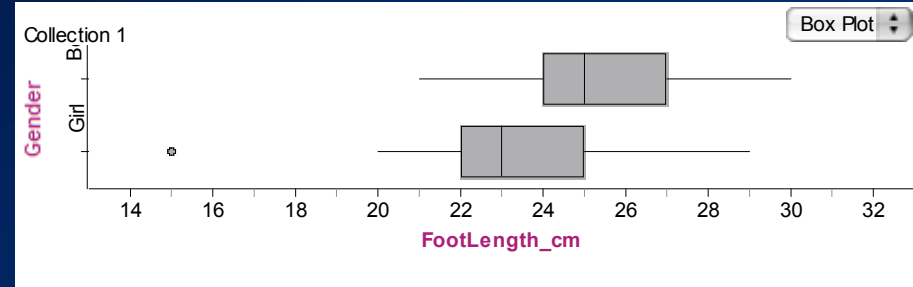
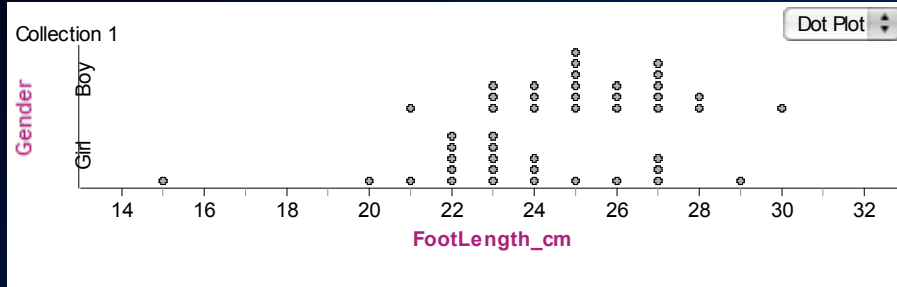
- the sample distribution for the boys' foot lengths is roughly symmetrical with a mound around 24 to 27cm, i.e., unimodal
- the sample distribution for the girls' foot lengths shows a large mound around 22 to 24 cm and a hint of a small mound around 27cm, i.e., a hint of bimodality

I wonder (*inferential*): :

- if boys' and girls' foot length distributions **back in the two populations** are roughly symmetric and unimodal.
 - I expect so for a body measurement such as foot length for both girls and boys.



Sampling

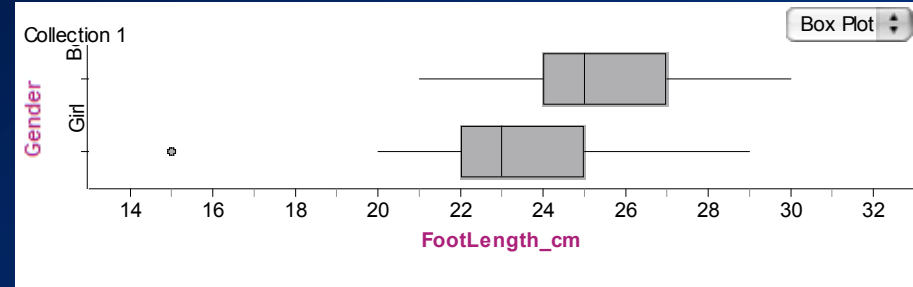
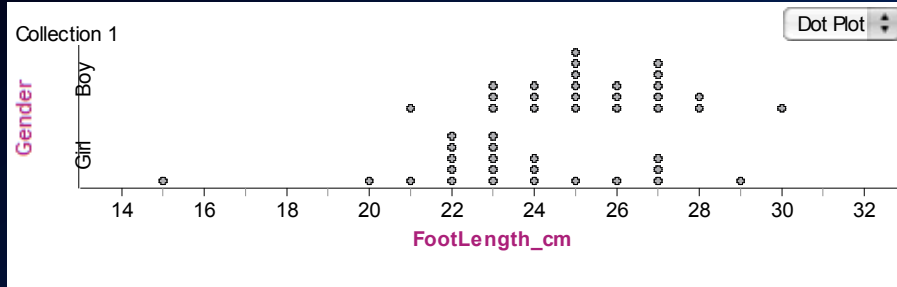


I wonder (*inferential*):

- if I repeated this sampling process many times the boys' foot lengths would, just about always, be shifted further up the scale than the girls'
- if boys tend to have a greater foot length than girls back in the two populations
- if the median foot length of boys really is greater than that of girls back in the two populations



Sampling - making the call - Level 5



What's my call?

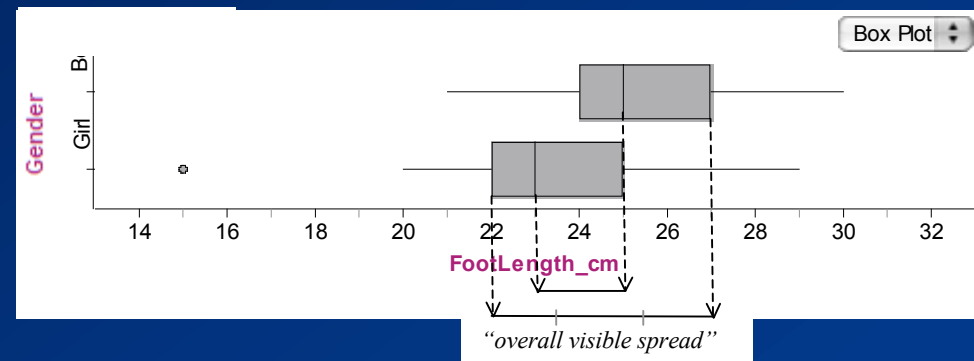
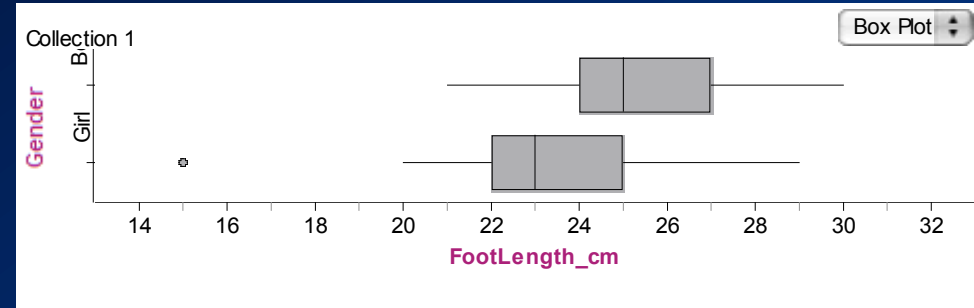
Am I prepared to claim that this pattern in the data is real, i.e., persists back in the two populations? Or could it have happened by chance?

Both sample sizes are between 20 and 40...



Sampling - making the call - Level 6

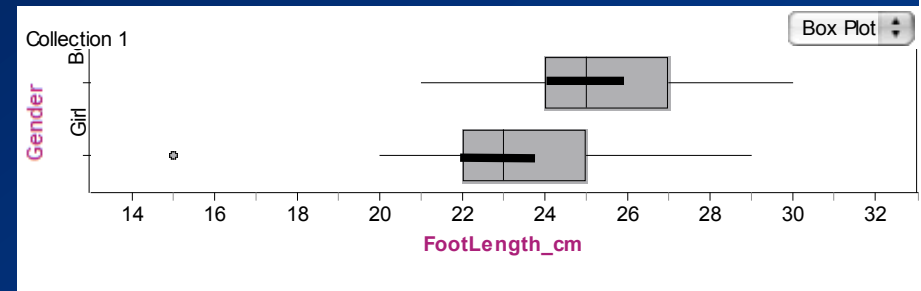
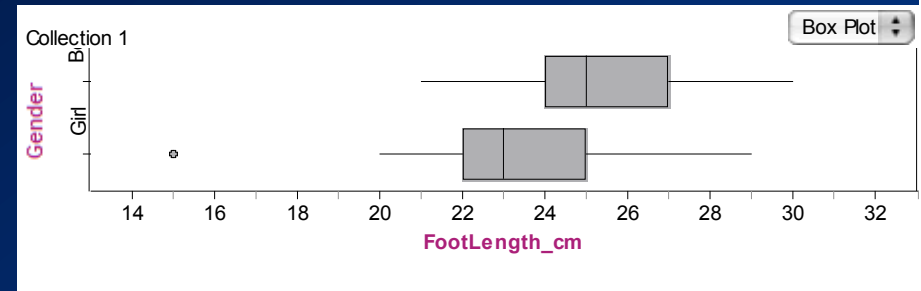
- What's my call?
- Distance between the boys' and the girls' median foot lengths is big relative to the overall visible spread...





Sampling - making the call -Level 7

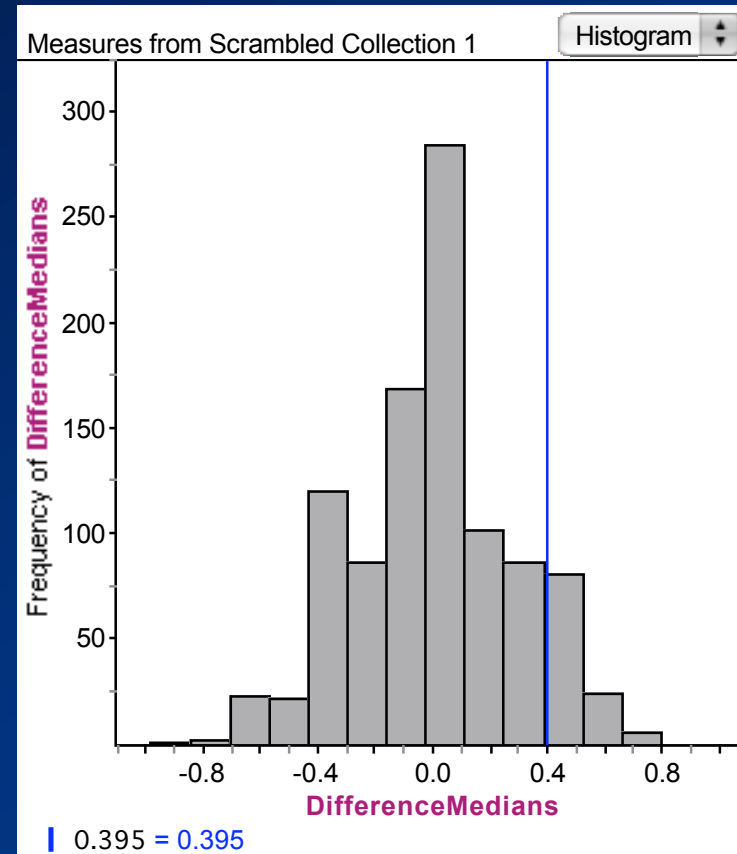
- What's my call?
- ... possible values for boys' and girls' population medians are somewhere within their respective informal confidence intervals ...





Sampling - making the call -Level 8

- What's my call?
- Quantifying the variation
 - 95% confidence intervals
 - Resampling method





Informal statistical inference

- Building statistical concepts - provide a pathway through the curriculum for students to understand the logic of inference
- See: www.censusatschool.org.nz/2008/informal-inference/

