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the NEW ZEALAND STATISTICIAN

CONFERENCE ISSUE

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THE NEW ZEALAND STATISTICIAN

Published by the New Zealand Statistical Association (Inc.)
Editor: R.B. Davies

Volume 8 June, 1973 No.2

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NOTICES

Annual Conference

The twenty-fourth annual conference of the New Zealand Statistical Association will be held in the Shell Theatrette, Shell House, The Terrace, Wellington, on Tuesday and Wednesday, the 3rd and 4th of July. The program and the abstracts of the papers appear elsewhere in this issue.

Annual General Meeting

This will be held in conjunction with the conference at 11.30 a.m. on Wednesday, 4th of July, in the Shell Theatrette.

Notice to Corporate Members

Corporate members of the association are invited to send representatives to the 1973 Annual Conference. However any such representatives who are not ordinary members are not permitted to vote at the A.G.M. except in the election of the corporate representative on the Executive Committee. The corporate representative, who must also be an ordinary member of the Association, is elected by the official representatives of the corporate members present (one representative per corporate member, the official representatives need not be ordinary members).

Statistics Seminar

Conference delegates are invited to a statistics seminar, organized by the Applied Maths Division of the D.S.I.R., to be held on Thursday, July 5th, 9–12.30 and 2–4.30, in room 730 of the Rankine Brown Building, Victoria University of Wellington. The speakers will be Mr B. Easton of the Economics Dept., University of Canterbury; Dr V.J. Thomas of the Applied Maths Division, Massey; Dr G.H. Jowett of the Invermay Agricultural Research Centre; Dr C.S. Withers and Dr S.F.L. Gallot of the Applied Maths Division, Wellington.

Calculator Display

The Association's display of electronic calculators takes place in the Lion and Unicorn rooms of the St George Hotel, Willis Street, Wellington, on Thursday, July 5th, 10 a.m. to 4 p.m. The display will be formally opened at 10.30 a.m. by the Hon. W.E. Rowling. The following firms are exhibiting:

Amalgamated Wireless (Australasia) NZ Ltd	(Kovac, Unitrex)
Armstrong & Springhall Ltd	(Olivetti)
Autocrat Business Machines	(Sanyo)
Beechey & Underwood Ltd	(Facit)
British Office Supplies NZ Ltd	(Ricomac)
Whitcoulls Ltd	(Hitachi)

EMI Industries (NZ) Ltd	(Sharp)
Fisher & Paykel Ltd	(Panasonic)
Fletcher Health and Science	(Wang)
Hewlett Packard NZ Ltd	(Hewlett Packard)
Kerridge-Odeon Canola Division	(Canon-Canola)
Pacific Business Systems	(Monroe)
"Silver-Reed" Electronic Calculator Centre	(Silver-Reed)

Model Answers

This year the Association is bringing out Model Answers for the 1972 T.C.A. examinations for the subjects: Mathematics III, Elements of Statistics, Systems and Data Analysis, and Applied Statistics II. They will be available shortly, the four papers being sold separately for 25 cents each. We ask members to bring this to the attention of all interested.

The New Zealand Statistician

The next issue of the **Statistician** is to be issued in October. Copy should be in by the first of September. Send it to –

The Editor,
New Zealand Statistician,
P.O. Box 1731
Wellington.

Subscription Rate for Libraries

Libraries may subscribe to the **New Zealand Statistician** at the rate of $4.00 per year. Subscriptions should be sent to –

The Business Editor,
New Zealand Statistician,
P.O. Box 1731, Wellington.

Correspondence

The association has its own post office box. Correspondence should be sent to –

The Secretary,
N.Z. Statistical Association,
P.O. Box 1731,
Wellington.

EDITORIAL

The New Zealand Statistician

In this editorial, I wish to comment on the purpose of the **New Zealand Statistician,** the types of articles which I consider suitable, and finally to make a few remarks to authors.

First, however, I should comment on the readership. If one looks at the Association's membership list one is struck by the wide range of members. There are the statisticians and biometricians (90) and those in closely related fields (50). But also there are quite a number (40) whose major interest is not closely related to statistics but who, presumably, from time to time need to use statistical methods. There are also a number of teachers (15) and a few students. The corporate membership of 38 represents a further group of statistics users. The readership is almost entirely confined to this country.

The purpose of the **Statistician** is to provide a means of communication between the members of the association: it is not to fill up library shelves or prevent academics from perishing. This means not only communication between people with similar interests but more importantly between the differing groups that make up the Association; in particular between the experienced practitioner and the hesitant user from some other field or between the theorist and the practitioner and between all of these and the teacher. I am particularly hopeful that the **Statistician** will encourage the use of statistical methods, where these are appropriate, by providing examples and by helping users and experts in various branches of statistics to contact each other.

While papers published in the **Statistician** must, of course, be of a high standard, highly specialized or technical papers will not, in general, be accepted – the readership for such articles would be very small and also there would be printing problems. However an author's abstract of a paper being published elsewhere would be very acceptable. This should be about one page (500 words) long. Also review or teaching papers in areas of an author's interest are useful to users, teachers, and statisticians. On the other hand, of particular interest are examples of practical applications of statistical methods, both as examples for teachers and users, and for suggesting problems to the theorists. Also acceptable are book reviews, letters to the editor, descriptions of general computer programs, short papers of general interest.

Now a few comments for authors. The **Statistician** is set by commercial typists using standard keyboards with no special mathematical symbols: summation and integration signs and Greek letters must be drawn in by hand and so should be avoided where possible. Mathematical formulae should be presented exactly as they should be produced with the usual conventions to simplify typing. Formulae which do not fit conveniently on one line must be displayed and plenty of space should be left around these. Papers should be in a format similar to that in recent editions and manuscripts should be double spaced with wide margins. Where possible figures should be in a form suitable for direct reproduction. It would be appreciated if papers and particularly abstracts involving numerous formulae or tables were submitted in a form also suitable for direct reproduction.

Finally may I express the hope that the previous paragraph has been addressed to a non-empty set of readers.

R.B.D.

TWENTY-FOURTH ANNUAL CONFERENCE

Shell Theatrette, The Terrace, Wellington

3 – 4th July, 1973

PROGRAM

Tuesday 3rd July:

9.30 a.m.	Mr R.J. Latimer	–	"Overseas Payments Forecasting".
11.00 a.m.	Mr M.A. Lumsden	–	"The Reserve Bank's Econometric Model".
2.00 p.m.	Mr B.D. Murphy	–	"The Statistical Basis of Opinion Polling".
3.30 p.m.	Mr N. Mountier	–	"Compartmental Analysis: A Biometrical Technique".
5.00 p.m.	Cocktail Party		

Wednesday 4th July:

9.00 a.m.	Mr F. Foster	–	"Medical Health Statistics Centre".
10.30 a.m.	Mr B.H. Easton	–	"Income Distribution".
11.30 a.m.	Annual General Meeting		
2.00 p.m.	Prof. D. Vere-Jones	–	"Statistical Estimation of Earthquake Risk".
3.30 p.m.	Dr G.H. Jowett	–	"The Loading Twist".

ANNUAL GENERAL MEETING

11.30 a.m. 4th July — Shell Theatrette

AGENDA

1. Apologies.
2. Confirmation of Minutes of 1972 Annual General Meeting.
3. Matters arising therefrom.
4. Presidential Report.
5. Treasurer's Report
6. Subscriptions: The Committee recommends that the subscription be raised to $2.00 for ordinary membership (with a 50 percent rebate for students) and remain at $8.00 for Corporate Membership.
7. Election of Officers (President, Secretary, Treasurer, 4 Committee Members, Corporate Representative and Auditor).
8. General.

ACKNOWLEDGEMENT

Yet again the Committee wishes to thank Shell Oil (NZ) Ltd for allowing the Association free use of the Theatrette, Shell House, for the Annual Conference.

NEW ZEALAND STATISTICAL ASSOCIATION

PRESIDENT'S REPORT FOR THE YEAR ENDED 31 MARCH 1973

Membership

There were 211 ordinary and 36 corporate members of the Association as at 31.3.73, as against 145 and 10 respectively at 31.3.72. The increase in corporate members resulted from the sending of letters to a number of business firms and Government departments pointing out to them the benefits to the country in general, and to them in particular, of their joining. We are grateful to those firms who subscribed to membership, and to the existing ones for their continued support. The increase in ordinary membership has been due to increased activity on the part of the Association, as reported below, but particularly to the publication of "Model Answers No.1". This was advertised in both the "Education Gazette" and "NZ Mathematics Magazine" and as a result a number of teachers joined. We now have 45 members who are employed in secondary or tertiary educational institutions. Membership has also widened to take in more people from the fields of social science and business, and now has a much broader base than in earlier years, when it was heavily biased towards census statistics and agricultural and associated sciences.

Committee

The committee met 6 times during the year. Officers for 1972-73 were:

President:	H.R. Thompson (D.S.I.R.)
Secretary:	T.O'H. Papps (Stats.)
Treasurer:	H.S. Roberts (D.S.I.R.)
Committee:	J.H. Maindonald (V.U.W.)
	S.S.R. Kuzmicich (Stats.)
	H. Offenberger (Polytech.)
	K.J.A. Revfeim (Dept. of Ag.)
	B.H. Easton (Univ. of Cant. - co-opted)
Corporate Members Rep.:	C.W. Walker (Shell)
Editor:	R.B. Davies (D.S.I.R.)
Hon. Auditor:	O.J. Ball

Conference

The Annual Conference was held at the Shell Theatrette on June 20 and 21, 1972. Papers were presented on a wide range of applied topics – economic planning models, market research, transport, educational research and recidivism – as well as on methodological techniques. The conference and cocktail party were both well attended. The Association is grateful to Shell Oil (NZ) Ltd for allowing us to use the Theatrette free of charge.

Evening Meetings

As a result of the lack of response from the Operational Research Society regarding the holding of regular monthly meetings, the Committee decided to investigate the feasibility of holding quarterly meetings of the Association.

Regional Representation

During the year Mr Brian Easton (Canterbury University) was co-opted on to the Management Committee in the capacity of unofficial Christchurch representative, and given a grant of $25 to support Association activities there. The Committee has been endeavouring to recruit a similarly enthusiastic person in the Auckland area, where there are also several members.

Calculator Display

Eleven firms participated in a display of electronic desk calculators which was arranged by the Association and held in the Hotel St. George on October 18, 1972. The display, which was opened by the Hon. R.D. Muldoon, was visited by about 500 people, and can be regarded as highly successful on the score both of publicity for the firms and the Association, and of finance, the net profit being over $400. As of the time of writing this report, we intend holding another display this year, on July 5, but we cannot guarantee that this activity will continue and thus become a regular source of income.

Model Answers

The Committee went ahead with publishing sets of model answers to the 1970 Technicians Certification Authority Certificate in Statistics examination papers, and on the basis of information received from institutions had 1000 copies printed. This was an over estimate and 600 copies remain unsold, as shown in the balance sheet. However, these should gradually sell over the years, and a side benefit of the project has been recruitment of many secondary school teachers to the Association. In the light of experience a smaller number of the 1972 papers is being printed for sale this year.

"New Zealand Statistician"

Three issues were published during the year. These contained a total of nine papers as well as notices, conference reports and abstracts, the membership list and calculator appraisals. It is proposed that there be three issues again in the coming year, and our Editor, Robert Davies (to whom our thanks are due for his sterling work) hopes that suitable articles will be forthcoming.

Expenditure on the "Statistician" is now our biggest item, and will almost certainly continue to be. The cost per copy is approaching $1, and it is unlikely that this will lessen. However, we feel that it is as important as any of our activities and ought to be continued, but this certainly cannot be done out of current subscription rates for individual members. For receiving the "Statistician" alone a subscription rate of $4 per year has been fixed and all major libraries circularised. So far five subscriptions have been taken up. However there are some organisations who have joined as corporate members solely

for the purpose of receiving the journal.

Finance

Cash reserves are down by $157, from $625 last year to $468 this year. Profit on the display was more than offset by costs of the "Statistician" and "Model Answers". As a means of keeping the Association financially stable, and allowing us to carry on our other worthwhile activities, the Committee recommends that the individual subscription be raised to $2 per member. This is still a very small sum for a professional body, and is of course also tax deductible.

Acknowledgement

In conclusion I would like to thank all Committee members for their support, and in particular a strong candidate for the title "Mr Statistical Association", Stan Roberts, who after many years of outstanding service to the Association is not available for re-election as treasurer.

H.R. Thompson

President.

FINANCIAL REPORT

FOR YEAR ENDED 31 MARCH, 1973.

FINANCIAL STATEMENT FOR THE YEAR ENDED 31st MARCH 1973

RECEIPTS & PAYMENTS

Receipts

Balance at 1.4.72			
In Hand	6.00		
In Bank	619.05		625.05
Subscriptions			
Individual	202.00		
Corporate (36)	288.00	490.00	
Annual Conference			
Cocktail Party	31.70		
Profit on Teas	1.09	32.79	
NZ Statistician			
Subscriptions	4.00		
Advertisements	30.00	34.00	
Calculator Display		550.00	
Model Answers		149.85	
Interest (BNZ)		12.87	
Donation		1.00	1270.51
			$1895.56

Payments

Annual Conference			
Guest Speaker			
(Travel)	54.00		
Hall	10.00		
Cocktail Party	42.69		106.69
NZ Statistician			
Vol.7 (1)	151.40		
(2)	144.70		
(3)	338.50		
Postage etc.	35.00		669.60
Calculator Display			
Hire of Room	69.30		
Printing	48.30		
Miscellaneous	3.84		121.44
Model Answers			
Typing	45.00		
Printing	340.00		
Advt.	15.00		
Postage etc.	25.00		425.00
Stamps, Stationery			55.27
Miscellaneous			49.34
			1427.34
Balance at 31.3.73			
In Hand	3.55		
In Bank	464.67		468.22
			$1895.56

INCOME AND EXPENDITURE ACCOUNT

Expenditure			Income	
Annual Conference		73.90	Subscriptions and Donations	491.00
NZ Statistician		635.60	Calculator Display	428.56
Model Answers			Interest (Bank)	12.87
Cost of answers sold	185.00			
Less sales	149.85			
Loss on answers sold		35.15		
General		104.61		
Excess Income over Expenditure		83.17		
		$932.43		$932.43

BALANCE SHEET

as at 31 March 1973

Liabilities		Assets	
Accumulated Funds (1.4.72)	625.05	Cash at Bank and on Hand	468.22
Excess Income over Expenditure	83.17	Stocks of Model Answers at cost	240.00
	$708.22		$708.22

H.S. Roberts
Treasurer

AUDITOR'S REPORT

To the Members of the New Zealand Statistical Association (Inc.) —

I have examined the Receipts and Payments Account and Income and Expenditure Account for the year ended 31 March 1973 and the Balance Sheet as at that date, and I have received all the explanations I have required. In my opinion proper books of account have been kept and the accounts and Balance Sheet give a true and fair view of the results for the year and the financial position of the Association.

O.J. Ball, A.C.A.

ABSTRACTS OF PAPERS

OVERSEAS PAYMENTS FORECASTING

Mr R.J. Latimer, Dept. of Statistics

Forward estimation of Overseas Payments for imports using distributed lags.

A stream of orders gives rise to a stream of payments. In some instances an order will be accompanied by payment but mostly importing is either in the hands of established importers with a good credit history and/or payment is in some other way guaranteed to the satisfaction of the supplier so that payment is not required before the delivery of the goods and in some cases not until some time later.

Many factors affect the lag between orders and payments. Even for a single commodity from a single source the lag is not determinate. If goods ordered cannot be supplied from stock there is an indeterminate delay while they are manufactured, then an indeterminate delay awaiting shipping and once shipped some uncertainty as to the time which will elapse before they are delivered, and there can even be some uncertainty as to how the time of payment will relate to the time of delivery.

For orders covering many commodities and many suppliers and many supplying countries it is necessary to describe the lags in statistical terms and to work in terms of mean lags and the distribution of lags. It is with the determination of the lag distributions and their use by the Department of Statistics in the projection of import payments that this paper is concerned.

THE RESERVE BANK'S ECONOMETRIC MODEL

Mr M.A. Lumsden, Reserve Bank

This paper presents a basically non-technical overview of what the Reserve Bank's econometric model is, what it is used for, and how it is built and operated. The model is a system of 95 simultaneous equations estimated by multiple regression methods using quarterly time series data from 1960 to 1971. This system of equations contains what are thought to be the most important causal linkages between various sectors of the economy. The model is used through simulation procedures to generate forecasts of the short-term future and to derive insight into how various government policy measures can or can not solve particular economic problems.

THE STATISTICAL BASIS OF OPINION POLLING

Mr B.D. Murphy, National Research Bureau

The NRB has been conducting opinion polls in New Zealand on a National basis since 1969 and is the only research organisation in the country doing polls regularly and publishing their results. Since November 1970 polls have been conducted in conjunction with the NRB bimonthly omnibus survey in which a sample of 1200 males and 1200 females is interviewed.

The sampe is selected by a random multi-stage method . In the first stage interviews are allocated to the urban centre and the surrounding rural country of the 19 distinct population areas in N.Z. in accordance with each area's share of population and the urban/rural population distribution in each area. Next, starting points for 15 interview clusters are determined using map grid references and street directories. The next stage is to call at every third house along a random route using a three call back system and to select a male and female in each house contacted by a random selection method.

Weightings are introduced in the analysis phase to remove any minor differences between the demographic characteristics of the sample and the total population. The representativeness of the samples has been consistently validated as has been their ability to measure known facts.

COMPARTMENTAL ANALYSIS: A BIOMETRIC TECHNIQUE

Mr N.S. Mountier, Lincoln College

Model building has become increasingly prominent in biology over recent years. The computer has allowed us to deal with far more complex models than anyone previously contemplated, and over the same period there has been an increasing emphasis on considering a system as a whole. Compartmental analysis is a modeling technique that has been developed to deal more adequately with metabolic processes in living systems. The model is defined in terms of a series of simultaneous equations which represent flows between compartments of the system. An example is given in which thyroxine in sheep is studied with the aid of a radio-active tracer.

NATIONAL HEALTH STATISTICS CENTRE

Mr F. Foster, Dept. of Health

Topics:

Outline of development of health statistics in New Zealand;

Range of data collected;

Relationship between the Centre and the users of health statistics;

Development of performance indicators;

Development of health resources data;

Participation in training statistical reporters;

Collaboration with international statistical agencies.

INCOME DISTRIBUTION

Mr B.H. Easton, University of Canterbury

This is a report on a study on the New Zealand Income Distribution since the second world war. Data to be presented will include measures of aggregate factor shares, the wealth distribution, dispersion of occupational incomes, and some measures of poverty levels. One of the objects of the study is to test the hypothesis that as New Zealand evolves the Income Distribution will become progressively less egalitarian.

STATISTICAL ESTIMATION OF EARTHQUAKE RISK

Prof. D. Vere-Jones, Victoria University of Wellington

Natural hazards in general, and earthquakes in particular, remain a very costly and troublesome aspect of our environment, whether the cost is measured in economics or in human terms. What aspects of such hazards are amenable to statistical treatment? Can statistical methods help in controlling or predicting earthquakes? Is there a statistical basis for insurance against earthquake?

None of these questions have an obvious answer, and despite the characteristically random pattern of earthquake occurence the application of statistical methods is beset with practical and theoretical difficulties. The paper will present a general discussion of these points, with particular emphasis on the problem of defining and estimating earthquake risk.

THE LOADING TWIST

Dr G.H. Jowett, Invermay Agricultural Research Centre

Faced with unpredictably varied jobs for which suitable programs are not to hand on the computer to which he has convenient access, the statistician can still make progress by applying his ingenuity to using well-tried standard programs in unusual ways. Thus, for example, most of the work for the rarely-needed multiple regression through the origin can be done by loading the data twice, once positively and once negatively, and then using a standard regression program. Most of the labour of analysis of the experiments of varied and often unorthodox design that seem to occur in agricultural research at Invermay is being done by applying a standard factorial analysis of variance program to data loaded as described in the talk.

A COMPARISON OF FOUR PROGRAMMABLE CALCULATORS

J.H. Maindonald

Victoria University of Wellington

and

R.M. Cassie

University of Auckland

In order to compare the 'real' programming capacity (from the point of view of the statistician) of four programmable calculators in the $1000 price range a 'statistical' type of calculation was programmed on each machine and the numbers of unused memories and program steps noted.

The calculators here considered are the Sobax ICC-2550E, the Canola 167P-II, the Burroughs C3660, and the Sharp Compet CS364PI. (c.f. also the Sharp Compet CS364PIII.) Other machines which fall into a broadly similar category are the Toshiba 1491GR and several machines in the Wang range. Note also the Sobax ICC-2700E and the Canola 1614P which are superior versions of our machines. No attempt is made to recommend a 'best buy'; much depends on the particular application.

For each machine a program has been devised which will calculate the sample correlation coefficient between $\log_e x$ and y. In program operation the data points (x,y) are entered in turn; and totals cumulated as follows :

$\sum \log_e x$ (Memory 1) $\quad\quad$ $\sum y$ (Memory 2)

$\sum (\log_e x)^2$ (Memory 3) $\quad\quad$ $\sum y^2$ (Memory 4)

$\sum (\log_e x)y$ (Memory 5)

n (or -n for the Sobax program) (Memory 6)

$\log_e x$ is in each case evaluated using the approximation

$$\log_e x = 2^6 \left(\sqrt[128]{x} - 1 / \sqrt[128]{x} \right)$$

If one is only interested in the correlation coefficient the factor of 2^6 may be omitted. We have preferred to leave it in.

When all data points have been entered program control is transferred to the final part of the program which evaluates the sample correlation coefficient.

Program for the Burroughs C3660

In place of <> we write RM, and SQRT rather than ÷ , when appropriate.

1	2	3	4	5	6	7	8	9	10	
*	.	EJ	8	H	SQRT	+=	SQRT	+=	SQRT	10
+=	SQRT	+=	SQRT	+=	SQRT	+=	SQRT	+=	÷	20
1	RC	-=	RC	-=	X	6	4	M+	1	30
X	M+	3	RC	X	H	RC	M+	5	RC	40
M+	2	X	M+	4	1	M+	6	J	8	50
EP	EP	RM	1	÷	RM	6	X	RM	1	60
M-	3	RC	X	RM	2	M-	5	RM	2	70
÷	RM	6	X	RM	2	M-	4	RM	4	80
X	RM	3	SQRT	+=	÷	RM	5	RC	+=	90
EP										

Program for the Canola 167P-II

1	2	3	4	5	6	7	8	9	10	
CM1	CM2	CM3	CM4	CM5	CM6	FJ	0	SQRT	SQRT	10
SQRT	SQRT	SQRT	SQRT	SQRT	÷	1	RV	-=	RV	20
-=	X	6	4	+=	M1	X	+=	M3	RV	30
X	E	M2	RV	+=	M5	RV	X	+=	M4	40
1	M6	EJ	0	RM1	X	+=	÷	RM6	-=	50
M3	RM1	X	RM2	÷	RM6	-=	M5	RM2	X	60
+=	÷	RM6	-=	M4	RM3	X	RM4	+=	SQRT	70
÷	RM5	RV	+=	E						

Program for the Sobax ICC-2550E

1	2	3	4	5	6	7	8	9	10	
M	CA	M	J	1	S	J	2	SQRT	SQRT	10
SQRT	SQRT	SQRT	SQRT	SQRT	÷	R	÷	R	-	20
R	R	X	()	6	4	()	=	M	1	30
Min	X	R	=	M	3	Min	R	X	S	40
M	2	Min	=	M	5	Min	R	X	R	50
=	M	4	Min	()	1	SIGN	()	M	6	60
Min	J	1	M	J	2	Mout	÷	M	1	70
Mout	R	X	Mout	R	=	M	3	Min	R	80
X	M	2	Mout	=	M	5	Min	R	X	90
R	÷	M	6	Mout	=	M	4	Min	Mout	100
÷	M	5	Mout	R	X	Mout	÷	M	3	110

Mout = SQRT END

Program for the Burroughs C3660

Set decimal point indicator to at least 6. There seems to be no particular advantage in setting it to F. For numbers in the range 10^{-4} to 10^{4} logarithms are then calculated accurate to approximately three significant figures.

After entering the program switch to AUTO mode, press C, and then press += to initiate the program. The program halts at step 5 for the entry of the first value of x which is followed by depression of +=. When program again halts at step 36 enter first value of y and depress +=, etc. When all data points have been entered switch to ALT/PGM and press +=. Program control then goes past the two EP instructions at steps 51-52 to step 53, leading to the evaluation of r and termination of the program.

Four memories and 53 program steps are unused.

Program for the Canola 167P-II

Decimal point setting and accuracy are as for the Burroughs.

After entering the program switch to OPE mode, then enter first value of x followed by START, etc. The first value of x is entered at step 1; subsequent values of x are entered at step 43. When in due course START is pressed without entry of a further value of x, program control goes from steps 43-44 to step 45 instead of looping back to steps 7-8. This leads to the evaluation of r and termination of the program.

The E program halt at step 75 could equally well appear as step 1. If this is done START must be pressed before the first value of x is entered. In programming from the keyboard memory operations for memories 3-7 require a second key stroke, but only one program step is counted.

One memory and 55 program steps are unused.

Program for the Sobax ICC-2550E

A decimal place setting of at least 6 is required; accuracy is then as for the other machines.

After entering the program switch to AUTO mode and press CA to bring program control to step 1. Press S to initiate program; the program halts at step 6 for entry of first value of x and depression of S to restart the program. When program again halts at step 40, enter first value of y and press S, etc. When all data points have been entered press S without entry of a further value of x. The negative display will cause transfer of program control to M J 2 at steps 64-66, leading to the evaluation of r and termination of the program.

Three memories and 11 program steps are unused.

Some Details regarding the Sharp Compet CS-364PI

We note features which differ from those on the Burroughs C3660. J→ is 'unconditional jump'; Jx<0 is 'jump on negative display'. In each case one of 0, 1, ..., 9, ., (+) is used to label the jump. A 'Jump on non-zero display' (Jx≠0) is available. The'recall and clear memory' (*) (memory total) is replaced by a 'clear memory' (CM) function which clears without recalling, and leaves the display unchanged. 'Clear all memories' is CM C ; 'clear memory 1' is CM 1. Also available is x→M, a 'store to memory' function which writes over anything there previously. In contrast with the M+ and M- keys, it does not function as +=. The program can be divided into any number of subprograms, which are separated from each other by two END's. Depression of the PS key followed by the number, in sequential order, of the required subprogram, transfers control to that subprogram. PS 1 has the same effect as the Sharp function CA except that the working registers are not cleared.

The decimal place/floating point setting may be varied between registers and memories.

The Burroughs program will run without modification on the CS364PI providing symbols are interpreted as follows :

Replace * . by CM C (not always equivalent)
EJ by JR (terminating point of jump)
RM (or <>) by MR
J by J→ (unconditional jump)
EP by END

After program has been entered switch to A (Auto), press CA, and then press S to initiate the program. The program will stop at step 5 with H displayed, and the first value of x can be entered. Note that S (and not +=) is pressed after each entry of data.
When all data points have been entered, press PS 2 (to select the second subprogram) and then S, which will lead to the evaluation of r and termination of the program.

Six memories and 53 program steps are unused. Note the possibility of transferring data between memories and the magnetic card; this function may be programmed.

Additional Features on the Sharp CS-364PIII

This machine, also in the stated price range, has an additional 288 steps for storage of 1 or more (up to a total of 9) subroutines. Two END instructions separate off one subroutine from another. The instruction 'branch to subroutine n' is programmed J→SUB n; once the designated sub-

routine has been executed control returns to the next succeeding step of the main program. Note that 'branch to subroutine' may not appear at the very beginning of a program; depression of J→SUB 1 (or even J→SUB n, n = 2,...,9) with S showing at step 0 in PRO mode (PROGRAM or LEARN mode) causes subsequent steps to be interpreted as subroutine steps.

In the program we have given steps 6-28 could be replaced by the two steps J→SUB 1, with steps 6-28 entered as steps 1-23 (followed by = END END) of subroutine 1. An easy modification (replace J→SUB 1 by J→SUB n; n = 2,...,9) would then replace the logarithmic transformation by whatever transformation the n^{th} subroutine (if it exists) has been designated to accomplish.

Depression of S as a program step enters the SKIP or 'Do Nothing' instruction. It is convenient to program two SKIP instructions at points where one may later want to substitute a 'branch to subroutine'.

Time taken to evaluate $\log_e x$:

This is the major part of the calculation. Burroughs takes 2.7 sec. and Sharp 1.4 sec., whether the decimal point indicator is set to 6 or to F. Canola takes 2.5 sec. in the former case, 3.0 sec. in the latter. Sobax takes 0.8 sec. (It has no F setting). The major part of this time is taken in evaluating square roots.

Access to memories:

After program calculations are completed, memories 1-5 contain the sums and sums of squares and products about the mean. Thus memory 3 contains Σy and memory 4 contains $\Sigma(y - \bar{y})^2$. These may be accessed using the memory subtotal keys while in MANUAL mode (Burroughs, Sharp and Sobax) or in OPE mode (Canola). Burroughs also allows reading of memories in DBG and AUTO modes. Canola and Sharp allow reading in or out in any mode except LRN (Sharp PRO). Also Canola and Sharp allow manual calculations to be freely performed whenever the program is halted in AUTO mode (Canola OPE), providing this does not interfere with calculations currently in progress.

Program Storage:

All machines except Canola read programs to and from magnetic cards when required. Canola uses a punched card. A possibility for the Canola program is to put steps 9-25 on a separate card. If this card is omitted when the program is entered, the effect is to omit the logarithmic transformation.

Some basic operations. For SHARP (similar to BURROUGHS) see above.

OPERATION	BURROUGHS	CANOLA	SOBAX
Clear all memories (also clears display on Burroughs; the display can be retained by using * . RC)	* .	CM1 CM2, etc	M CA
Clear memory (e.g. memory 3) and store displayed value a	* 3 RC M+ 3	SM3 (3-7 only)	M 3 MC Min
Add to memory	M+ 3	M3	M 3 Min
Subtract from memory	M- 3	M-1 (1&2 only)	Use ()SIGN(),etc
Memory subtotal	<> 3	RM3	M 3 Mout
Memory total (clears memory)	* 3	RM3 CM3	M 3 Mout MC
Evaluate result (e.g. of a X b), and add to memory	M+ 3	+= M3	= M 3 Min
Constant among program steps	k	k	() k ()
Evaluate ab, recall a	a X b =, then RV	ditto, then RC	b X a =, then R
a ÷ b, recall b (-b is recalled after depression of -= on Burroughs & Canola)	a ÷ b =, then RV	ditto, then RC	ditto, then R

All machines interpret a X b R as b X a, and a ÷ b R as b ÷ a. (Here R ≡ RV ≡ RC). Sobax allows, in addition, a + b R and a - b R.

a^2	X +=	X +=	X R =
Square root of a	÷ +=	SQRT	SQRT

On Burroughs a X b X += gives $(ab)^2$. Canola gives a^2b for this; while Sobax a X b X = gives ab.

On Burroughs a X b X c ÷ += gives $(abc)^{\frac{1}{2}}$.

End of Program	EP	not needed	END
Terminal point for jump	EJ 3	FJ 3	M J 3
Jump on negative display (including 0 if noted)	J 3 (jumps on -0) (may replace 3 by 1,2, ... , 7 or H)	MJ 3	J 3 (jumps on 0)
Jump unconditionally	J 8 (may replace 8 by 9,0,SIGN or C)	UJ 3	(make display -ve and jump on -ve display)
Jump following halt if data entered	-	EJ 3	-
Sense jump (halt, jump if SJ pressed) -	-	SJ 3	-
Division of program into 2 parts	Use EP EP	Prog I & II	-

Note: C/ALL has been written as CA and CHG/SGN has been written as SIGN.

Operations involving more than 2 terms :

Write (a.e.) for arithmetic expression. On all machines (a.e.) X, or (a.e.) ÷ leads to evaluation of the arithmetic expression. The Sobax + and - also have this effect. On the Canola and Sobax one can use this to store the result of an intermediate calculation; thus

a X b X Min c =

stores ab in the last referenced memory and evaluates abc. A 'halt program' in place of Min would display ab, on all machines.

If = is followed by R (or RC or RV), c is displayed on the Sobax, ab on the other machines.

Note on the derivation of the approximation :

By subtracting the series approximation for log(1-u) from that for log(1+u), ignoring terms of degree 3 or higher, and setting a = (1+u)/(1-u), so that (a-1)/(a+1) = u, we get the approximation :

$$\log_e a = 2(a-1)/(a+1)$$

which is valid for values of a close to 1.

Now observe that :

$$(a-1)/a < 2(a-1)/(a+1) < a-1 ,$$

with the first and third terms both close in value to the second if a is close to 1. In this case the average of the first and third terms, i.e. $\frac{1}{2}((a-1)/a + a-1) = \frac{1}{2}(a - 1/a)$, will be a still closer approximation to the quantity 2(a-1)/(a+1) , which is an approximation to $\log_e a$.

The approximation we use is therefore $\log_e a = \frac{1}{2}(a - 1/a)$.

In order that a will be close to 1, we set $a = \sqrt[n]{x}$ and use

$$\log_e x = n \log_e \sqrt[n]{x}$$

Our formula used 7 square roots to give n = 128, $\frac{1}{2}n = 64$. The use of 8 square roots (n = 256) would give rather better accuracy; at least four significant figures in the range 10^{-4} to 10^{4}.

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Our thanks are due to Dr K J A Revfeim who suggested the approximation for $\log_e x$ which we finally used, and drew attention to the use of a ÷ R ÷ R - R as the best means of programming 1/a - a on the Sobax.